ANALYTICAL STUDY OF HEAT AND MASS TRANSFER IN THE HARDENING KINETICS OF CONCRETE HEAT-TREATED IN AN ELECTROMAGNETIC FIELD

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The results are given from an analytical and experimental study of the kinetics of the heat treatment of ferroconcrete slab structures in an alternating electromagnetic field.

A technique for accelerating the hardening of concretes in the wet heat treatment of ferroconcre structures in an industrial-frequency electromagnetic field is currently under investigation and in pract operation at the Institute of Heat and Mass Transfer of the Academy of Sciences of the Belorussian SSR [2].

The distinguishing feature of this heat-treatment process is the relatively uniform withdrawal of heat throughout the entire volume of the concrete due to the conversion of electrical energy into heat in ferromagnetic form and reinforcement matrix of the specimens, i.e., a virtually uniform distribution o both external and internal heat sources. Under the conditions of actual heat transfer with the surroundi medium, however, a temperature field is formed, which naturally induces the formation of a moisture-content field.

The electromagnetic heat treatment of ferroconcrete slabs in a metal form with a reinforcement matrix distributed uniformly throughout the concrete can be described by a set of heat- and mass-trans differential equations [3] with internal heat sources and moisture sinks. In the heat treatment of ferroconcrete slabs on a ferromagnetic substrate (metal form) the process can be described by the following set of differential equations (one-dimensional problem):

$$\frac{\partial t(x,\ t)}{\partial \tau} = a\,\frac{\partial^2 t(x,\ \tau)}{\partial x^2} + \frac{\varepsilon\rho}{c}\,\frac{\partial u(x,\ \tau)}{\partial \tau} + \frac{IW + q_2}{c\gamma},$$

$$\frac{\partial u(x,\ \tau)}{\partial \tau} = a_m\,\frac{\partial^2 u(x,\ \tau)}{\partial x^2} + a_m\delta\,\frac{\partial^2 t(x,\ \tau)}{\partial x^2} - \omega$$

subject to the boundary conditions

$$-\gamma_0 a_m\left[\frac{\partial u(0,\ \tau)}{\partial x} + \delta\,\frac{\partial t(0,\ \tau)}{\partial x}\right] = q_m,$$

$$\lambda\,\frac{\partial t(R,\ \tau)}{\partial x} + \alpha\,[t(R,\ \tau) - t_{\mathrm{cp}}] + \rho\,(1-\varepsilon)\,q_m = 0,$$

$$a_m\,\frac{\partial u(R,\ \tau)}{\partial x} + a_m\,\delta\,\frac{\partial t(R,\ \tau)}{\partial x} + \beta u\,[u(R,\ \tau) - u_p] = 0,$$

$$t(0,\ \tau) = f(\tau),$$

$$t_{\mathrm{av}} = f_1(\tau).$$

The initial conditions are as follows:

$$t(x,\ 0) = t_0;\ u(x,\ 0) = u_0.$$

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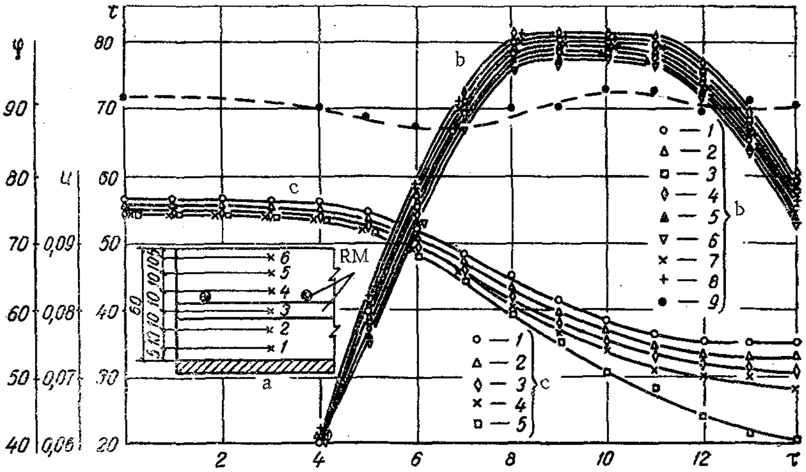

Fig. 1. Kinetic curves for the wet heat treatment of ferroconcrete slabs in an electromagnetic field. a) Schematic of the ferroconcrete specimen (layers numbered 1 through 6; RM) reinforcement matrix); b) variation of the temperature (°C) in the concrete and the percentage humidity (%) of the treatment-chamber atmosphere with time (h): 1) first (bottom) layer; 2) second layer; 3) third; 4) fourth; 5) fifth; 6) sixth (top) layer; 7) form temperature; 8) reinforcement matrix temperature; 9) percentage humidity of chamber atmosphere; c) local moisture distribution (kg/kg) in the concrete versus time (h): 1) first (bottom) layer); 2) second layer; 3) fourth; 4) fifth; 5) sixth (top) layer. So as to avoid error the moisture content of the third layer containing the reinforcement matrix was not determined.

In the present article we generalize and correlate the results of analytical and experimental investigations of the temperature and moisture-content field distributions in the hardening kinetics of concrete subjected to wet heat treatment, during the period in which the temperature rises to a maximum in the specimen, as it is in this part of the temperature regime (see Fig. 1) that the main physicochemical processes involved in the coagulation and crystallization structurization of the cement take place. In this period, as a rule, the destructive processes elicited by the temperature-humidity strains of concrete experience their peak development.

In Eq. (1) we have introduced the term $(IW + q_2)/c\gamma$ on the basis of the following consideration.

The hydration chemical reaction of cement in a hardening mass of concrete is accompanied by the uniformly volume-distributed internal release of heat from the cement. We denote this heat source by $q_1 = IW$, i.e., the power of the internal heat source formed by hardening of the concrete is equal to the product of the power W of the internal moisture sink due to losses in the molecular (hydrate) bond by the enthalpy I of the given system.

We know that the hydration of mineral binding agents is a strongly exothermal process. The total heat release determines the amount of chemically bound water, i.e., the degree of hydration of the cement (the relative completion of the chemical reactions of the cement minerals with water). As the quantity of water in the molecularly bound state is increased (according to the scheme of P. A. Rebinder), the heat of hydration increases in the crystal hydrates of cement, attaining, according to the data of [4], $(40\text{-}50) \cdot 10^3$ kcal/m^3 of concrete, which is equivalent to a temperature increase of 70-80°C. The maximum rate of heat release in this case is observed in the first two or three hours of hardening. The heat imparted to the system is spent in increasing the enthalpy I, which is a thermodynamic function of the state of the system. Now, by virtue of the equation $dI = dU + pdV$, this heat goes toward increasing the interval energy of the system and completing the work of increasing the volume of the new-formed cement.

Thus, according to Powers [5] the hydration of Portland klinker doubles the volume of the hard constituents in the cement mix, even though the process as a whole proceeds without significant volume changes of the specimens.

Next we represent the heat release in the reinforcement matrix, which is distributed uniformly in the concrete, by the source q_2. The total internal source is $Q = q_1 + q_2$. It must be noted at the same time that, owing to the fundamental dependence of the internal exothermy on the temperature [6, 7], this result will tend to smooth out the irregularity of the temperature field in the given heat-treatment technique.

Equation (2) includes the additional term $\omega = 100 \cdot W a_m / \lambda_m$, which accounts for the internal moisture sink due to water entering into chemical bond with the cement, resulting in the production of high-hydrate and low-hydrate modifications of the new-formed cement.

The right-hand side q_m of the boundary condition (3) is equal to zero, because for $x = 0$ the moisture flow in the direction of the metal form, which acts as a moisture insulator, is equal to zero.

The solution of the system (1)-(8) with linear variation of the temperature of the form and surrounding medium, taking thermal diffusion and moisture evaporation from the slab surface into account, presents considerable mathematical difficulties. The specific conditions of the wet heat treatment of ferroconcrete specimens in electromagnetic chambers permit the problem (1)-(8) to be simplified. Following are the principal assumptions.

Recognizing that when a ferroconcrete slab is heated in an electromagnetic field the temperature drop over its cross section in a given part of the regime is slight (see Fig. 1), plus the fact that the instantaneous moisture content is close to the initial value in this period, the value of the thermal gradient coefficient δ is small. In Eq. (2), therefore, the second term can be dropped.

Proceeding from the experimental data, we can assume that the phase conversion criterion ε in concrete [8] is equal to zero, since the transfer of moisture in vapor form is smaller by three orders of magnitude than that in liquid form. Consequently, we can neglect the second term of Eq. (1).

Moreover, in the concrete heating period the principal internal heat- and mass-transfer coefficients λ, a, c, and a_m are variables, due to the time variation of the physicochemical properties, temperature, and moisture content of the concrete. In view of the fact that the analytical solution of the heat- and mass-transfer system with time-variable thermophysical and mass-transfer characteristics also presents sizable difficulties, in order to calculate the moisture-content fields we arbitrarily partition the heating period into four equal time segments (zones). Now within the limits of each segment the values of the coefficients can be assumed constant but different in numerical value for each succeeding segment of the heating period.

Inasmuch as only a very minute quantity of moisture is evaporated from the concrete in the given part of the temperature regime due to the high relative humidity of the surrounding medium (φ = 85-95%), the heat of vaporization can be neglected.

The indicated internal heat source Q, according to the foregoing considerations, is assumed to be uniformly distributed throughout the slab volume. Thus, the mathematical statement of the problem is as follows for this case:

$$\frac{\partial t(x,\tau)}{\partial \tau} = a\frac{\partial^2 t(x,\tau)}{\partial x^2} + \frac{IW + q_2}{c\gamma}, \tag{9}$$

$$\frac{\partial u(x,\tau)}{\partial \tau} = a_m \frac{\partial^2 u(x,\tau)}{\partial x^2} - \omega, \tag{10}$$

$$\frac{\partial u(0,\tau)}{\partial x} = 0, \tag{11}$$

$$\gamma_0 a_m \frac{\partial u(R,\tau)}{\partial x} + q_m = 0, \tag{12}$$

$$\lambda\frac{\partial t(R,\tau)}{\partial x} + \alpha[t(R,\tau) - t_{av}] = 0. \tag{13}$$

Newton's law is taken into account in Eq. (13). Since a quasi-stationary regime is assumed to prevail, the heat-transfer coefficient α is presumed to be constant:

$$t(0,\tau) = A\tau + B;\ t_{av} = A_1\tau + B_1;\ t(x,0) = t_0, \tag{14}$$

$$u(x,0) = u_0. \tag{15}$$

TABLE 1. Initial Data for Calculation of the Temperature and Moisture-Content Fields

Time from start of heat treatment, h	Number of subinterval (zone) of heating period	R	γ	A	B	c	λ	$a\cdot10^4$	$a_m\cdot10^4$	u_e	u_0	h	ω	q	βu
1	I	0,06	2435	15	20	0,26	1,86	29,2	2,47	0,02	0,095	4,3	0,005	9800	0,00163
2	II	0,06	2420	15	20	0,25	1,78	29,2	4,55	0,02	0,095	4,48	0,00185	10200	0,00168
3	III	0,06	2410	15	20	0,25	1,72	28,5	6,93	0,02	0,095	4,65	0,001	10700	0,0034
4	IV	0,06	2400	15	20	0,24	1,65	28,5	11,3	0,02	0,095	4,85	0,0008	11300	0,0045

TABLE 2. Analytical and Experimental Values of the Temperature and Moisture-Content Field Distributions in Heat-Treatment Kinetics

Specimen thickness	Moisture content u, kg/kg vs time h								Temperature t, °C vs time h							
	1		2		3		4		1		2		3		4	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
$x=0$	0,0949	0,0940	0,0926	0,0925	0,0878	0,0890	0,0785	0,0850	35	39	50	56	65	69	80	80
$x=\frac{1}{4}R$	0,0936	0,0930	0,0922	0,0920	0,0855	0,0875	0,0768	0,0830	35	39	50	55	65	68	80	79
$x=\frac{1}{2}R$	0,0933	0,0925	0,0908	0,0900	0,0821	0,0860	0,0757	0,0815	35	38	50	53	65	68	80	81
$x=\frac{3}{4}R$	0,0929	0,0925	0,0903	0,0900	0,0837	0,0860	0,0745	0,0810	35	36	50	52	65	66	80	77
$x=R$	0,0900	0,0920	0,0878	0,0880	0,0804	0,0840	0,0730	0,0785	35	35	50	52	65	66	80	76

Note: A denote the analytical, B the experimental data.

The solution of problem (9), (13), (14) has the form

$$t(x,\ \tau) = A\tau + B + \frac{\left(\frac{IW + q_2}{c\gamma} - A\right)}{2a}\left[\frac{(2 + Rh)\,Rx}{1 + Rh} - x^2\right]$$

$$+ \frac{4\left(\frac{IW + q_2}{c\gamma} - A\right)hR^3}{a} \cdot \sum_{n=1}^{\infty} \frac{\sin\left(\frac{\mu_n}{R}x\right)(1 - \cos\mu_n)\,e^{-\frac{a\mu_n^2}{R^2}\tau}}{\mu_n^2 \sin 2\mu_n\,[Rh\,(1 + Rh) + \mu_n^2]}, \tag{16}$$

where μ_n represents the positive roots of the equation $\mu \cot \mu + \mathrm{Rh} = 0$.

Here, beginning at a certain time in the concrete heating period, the temperature difference $t(0,\ \tau) - t_{av}$ becomes insignificant and may therefore be neglected in Eq. (16) for the calculation of the temperature field. Under the condition $\omega = \mathrm{const}$, applying the Laplace transform to problem (10), (11), (12), (15), we have the following solution for the moisture-content field:

$$u(x,\ \tau) = \frac{R\omega}{\beta u} - \frac{R^2 - x^2}{2a_m}\omega + 2\sum_{k=1}^{\infty} \frac{\left(\frac{\omega R^2}{a_m\mu_k^2} + u_0 - u_e\right)\cos\left(\frac{x}{R}\mu_k\right)e^{-\frac{a_m\mu_k^2\tau}{R^2}}}{\mu_k\left[\left(1 + \frac{a_m}{R\beta_u}\right)\sin\mu_k + \frac{a_m\mu_k}{R\beta_u}\cos\mu_k\right]}, \tag{17}$$

in which μ_k represents the positive roots of the equation

$$\mathrm{ctg}\,\mu = \frac{a_m}{R\beta_u};\quad \beta_u = \frac{\mu}{\mathrm{Bi}_m}.$$

Under the condition $\omega = \omega(\tau)$, applying a finite integral transform to the same problem, we find the solution in the form

$$u(x,\ \tau) = u_e + 4\sum_{k=1}^{\infty} \frac{\left[u_0 - u_e - \int_0^{\tau} \omega(\eta)e^{\frac{a_m\mu_k^2\eta}{R^2}}d\eta\right]e^{-\frac{a_m\mu_k^2\tau}{R^2}}\sin\mu_k\cos\frac{\mu_k}{R}x}{2\mu_k + \sin 2\mu_k}, \tag{18}$$

where μ_k represents the positive roots of Eq. (17').

For each of the segments into which we arbitrarily partitioned the temperature-rise period the coefficients λ, a, and c have been obtained experimentally. The initial data for the calculation of the temperature and moisture-content fields are given in Table 1. Using the radiometric method, we experimentally determined the local moisture contents during the hardening of concrete under heat treatment by transmitting a collimated beam of thulium-170 isotope λ-quanta through each layer of the slab [9]. The moisture diffusion coefficient for each subinterval of the temperature-rise period was determined by the procedure of [10]. The value of the mass conductivity λ_m for concrete of the given composition was taken from [11].

The quantity W of chemically bound water was determined experimentally in the heat-treatment kinetics by the procedure of Butt [12]. The composition of the high-test concrete is given in [2].

The calculated values of the temperature and moisture-content field distributions in a ferroconcrete slab according to Eqs. (16) and (17) were compared with the experimental data (Table 2).

As apparent from Table 2, the results of the calculations according to the simplified procedure are in good agreement with the experimental. The solution of this problem can be extended to preformed ferroconcrete objects such as flat slabs and panels whose thickness is several times smaller than the other two dimensions (length and width).

Similar calculations can be carried out for long straight objects such as beams, columns, etc., by treating them as infinite cylinders having an equivalent cross section.

NOTATION

t	is the running temperature, °C;
u	is the running moisture content, kg/kg;
a	is the thermal diffusivity, m^2/h;
a_m	is the moisture diffusion coefficient, m^2/h;
λ	is the thermal conductivity, $kcal/m \cdot h \cdot deg$;
λ_m	is the mass conductivity, $kg/m \cdot h \cdot deg$;
c	is the specific heat, $kcal/kg \cdot deg$;
δ	is the thermal gradient coefficient, deg^{-1};
γ	is the density of the concrete, kg/m^3;
Q	is the total power of internal sources, $kcal/m^3 \cdot h$;
ε	is the dimensionless phase conversion coefficient;
ρ	is the specific heat of vaporization, kcal/kg;
γ_0	is the kiln-dry density of the concrete, kg/m^3;
α	is the heat-transfer coefficient, $kcal/m^2 \cdot h \cdot deg$;
q_m	is the moisture flux, $kg/m^2 \cdot h$;
β_u	is the mass-exchange coefficient, m/h;
A, B, A_1, B_1	are constant coefficients;
$h = \alpha/\lambda$, m^{-1};	
R	is a characteristic dimension, m;
u_e	is the equilibrium moisture content, kg/kg;
u_0	is the initial moisture content, kg/kg;
W	is the quantity of moisture entering into chemical bond with the cement per unit volume of concrete per unit time, $kg/m^3 \cdot h$;
I	is the system enthalpy, kcal/kg;
$Bi_m = \beta_m R/a_m$	is the Biot mass-transfer criterion.

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